Consideration of Magnetic Saturation in a New Hybrid Semi-Numerical Model

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The aim of this paper is first to briefly describe a mathematical approach for the direct coupling between analytical formal solution of Maxwell's equations (AM) and magnetic equivalent circuits (MEC) (reluctance networks or permeance networks) methods for the electromagnetic modeling of electric machines. This coupling can help solve the problem of air-gap modeling in MEC method, and the consideration of the local magnetic saturation in modeling approaches involving analytical formal solution of Maxwell's equations. The formal solution of Maxwell's equations is obtained by using separation of variable method along with Fourier series analysis; it is used to model constant permeability regions (magnetic air-gap). The MEC method is used to model regions which include ferromagnetic parts. MEC model is generated based on a mesh of the studied domain. Different approaches related to the consideration of magnetic saturation are investigated and compared.

***Index Terms*— Electromagnetic modeling, electromagnetic analysis, magnetic circuits, performance evaluation, Maxwell equations, finite element analysis, Magnetic saturation**

I. Introduction

THIS CONTRIBUTION investigates inclusion of magnetic saturation in a new hybrid semi-numerical method [1]–[4]. This new modeling approach is first briefly described. It is based on a direct coupling of analytical formal solution of Maxwell's equations (AM) and magnetic equivalent circuits (MEC) (reluctance networks or permeance networks) methods [5] [6].

While the formal solution of Maxwell equations is sought in constant permeability regions (mainly the magnetic air-gap), mesh based generated MEC method [5] [6] is used to model regions which include ferromagnetic parts subject to magnetic saturation.

At a first glance, magnetic saturation is always neglected in developed analytical models for electric machines, in machine's regions where the analytical solution is sought, as a consequence of adopted assumptions (linear magnetic materials or with infinite relative permeability). Examples where analytical models, based on the formal solution of Maxwell's equations, are involved and where the local magnetic saturation is taken into account are situations where the analytical models are either combined with finite element method [7], or coupled with magnetic equivalent circuit (MEC) method [1]–[4].

While, in [1] and [2], the authors developed a hybrid model where the analytical solution is based on magnetic vector potential, the approach adopted in this contribution is based on the analytical solution of magnetic scalar potential.

Different aspects related to the consideration of magnetic saturation in this new modelling approach are discussed. Two solving methods, i.e., successive approximation method and Newton-Raphson method, for the obtained non-linear equations system will be compared, in terms of computation effort. The comparison study will also include analysis of the effect of unknowns nature choice, for the MEC model, i.e., nodes magnetic scalar potential or branches flux, on the computation effort [8].

II. Consideration of the Magnetic Saturation

Before discussing the inclusion of the magnetic saturation, the new modelling approach is briefly described. Figure 1 illustrates how the two approaches are combined.

For this structure (Fig. 1), the general solution of Maxwell equations based on magnetic scalar potential formulation, in PM and mechanical air-gap regions, can be expressed for a region i by [4]

$$U^{(i)}(x,y) = a_0^{(i)} + a_1^{(i)} x + a_2^{(i)} y + a_3^{(i)} xy + \sum_{n=1}^{+\infty} \left[\begin{aligned} &\left(\begin{aligned} &a_{4n}^{(i)} \sinh(n\pi\, y/\tau_p) \\ &+ a_{5n}^{(i)} \cosh(n\pi\, y/\tau_p) \end{aligned} \right) \sin(n\pi\, x/\tau_p) \\ + &\left(\begin{aligned} &a_{6n}^{(i)} \sinh(n\pi\, y/\tau_p) \\ &+ a_{7n}^{(i)} \cosh(n\pi\, y/\tau_p) \end{aligned} \right) \cos(n\pi\, x/\tau_p) \end{aligned} \right] \quad (1)$$

Considering geometric and electromagnetic periodicities of studied structure (end effects are not considered), it is easy to demonstrate that $a_1^{(i)} = a_2^{(i)} = 0$ and $a_3^{(i)} = 0$, for both regions. $a_0^{(i)}$ is an arbitrary constant which can be set null.

Mesh-based generated MEC method has been used to model the stator iron core and slots. Although this approach is relatively old [5] [6] and gives relatively good results, it is still not widely spread. As finite element method, this approach consists of meshing the studied domain. Reluctance elements are used for the mesh.

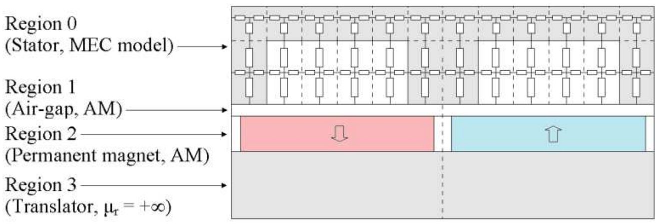

Fig. 1. Illustration of the modelling approach on a linear structure.

The coupling between the AM and the MEC model is done at the boundary between both models by equalising the tangential components of magnetic field ***H***, and normal components of magnetic flux density ***B*** [4].

$$\begin{cases} H_t(\mathrm{AM}) = H_t(\mathrm{MEC}) \\ B_n(\mathrm{AM}) = B_n(\mathrm{MEC}) \end{cases} \quad (2)$$

This model will result on a set of equations which can be expressed in matrix form

$$[\mathrm{A}][\mathrm{X}] = [\mathrm{E}] \quad (3)$$

where [A] is the topological matrix where the elements are depending on the geometrical shape of the limits between the different regions of the machine and magnetic properties of the different materials; [E] is the source vector, elements of which are related to geometry distribution and physical properties of magnetic field sources (magnetic remanence and current density distributions) and [X] is the unknowns vector whose are the series coefficients of the analytical solution of scalar potential (AM), and unknowns issued from the MEC model.

The consideration of magnetic saturation will only concern the equations issued from the MEC model in ferromagnetic regions. This set of non-linear equations is solved in an iterative way using classical methods: successive approximation method and Newton-Raphson method. More details will be provided in the full version of the contribution.

III. Validation

The developed model (hybrid model, HM) has been used to analyse the performance of a linear structure (Fig. 2). Figure 2 shows the studied structure, with main geometric dimensions. Table I gives main machine's characteristics. The stator back iron has been chosen very thin in order to amplify the magnetic saturation effect. This structure has also been modelled and analysed using FE method.

Figure 3 compares cogging force waveforms obtained by both methods with consideration of magnetic saturation. As can be seen relatively good agreement is obtained. The non-linear equations system has been solved using successive approximation method.

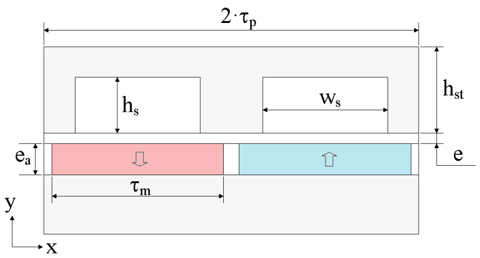

Fig. 2. Longitudinal cross section view of studied linear structure with main geometric dimensions.

TABLE I – Machines Parameters

e (mm)	1	τ_m (mm)	55
e_a (mm)	10	τ_p (mm)	60
h_s (mm)	25	w_s (mm)	40
h_{st} (mm)	30	PM remanence (T)	1.2

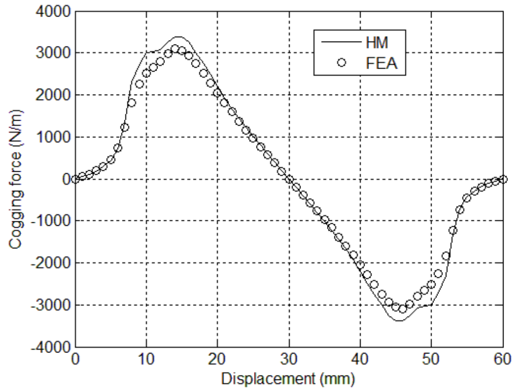

Fig. 3. Comparison of cogging force waveforms.

IV. Conclusion

A new modeling approach has been first described. This approach consists of directly coupling analytical models (AM) obtained through formal solution of Maxwell's equations, in constant permeability regions, and MEC models for regions including ferromagnetic materials. MEC model is generated by meshing the studied domain as for FE method. This technique helps combine advantages of both methods. It should allow time saving as compared to FE method and can be advantageously used in pre-design stages.

It has been shown, on a case study, that this technique is as effective as FE method for consideration of magnetic saturation. In the full version of the contribution more details will be provided. Different aspects on the implementation of this technique will be reported.

References

[1] H. Gholizad, M. Mirsalim, M. Mirzayee, "Dynamic analysis of highly saturated switched reluctance motors using coupled magnetic equivalent circuit and the analytical solution," in Proceedings of CEM 2006, International Conference on Computational Electromagnetics, Aachen, Germany, VDE, April, 2006, pp. 1–2.

[2] H. Ghoizad, M. Mirsalim, M. Mirzaei, W. Cheng, "Coupled magnetic equivalent circuits and the analytical solution in the air-gap of squirrel cage induction machines," International Journal of Applied Electromagnetics and Mechanics vol. 25, 2007, pp. 749–754.

[3] S. Ouagued, Y. Amara, G. Barakat, "Comparison of hybrid analytical modelling and reluctance network modelling for pre-design purposes," in Proceedings of ELECTRIMACS 2014, 19th-22nd May, 2014, Valencia, Spain.

[4] Y. Laoubi, M. Dhifli, G. Verez, Y. Amara, and G. Barakat, "Open circuit performance analysis of a permanent magnet linear machine using a new hybrid analytical model," to be published in *IEEE Trans. on Magnetics*.

[5] C. B. Rasmussen and E. Ritchie, "A magnetic equivalent circuit approach for predicting PM motor performance," in Proceedings of *IEEE. Industry Applications Society Annual Meeting*, vol. 1, no. 1, pp. 10-17, October 1997, New Orleans, Louisiana, USA.

[6] J. Perho, *Reluctance network for analysing induction machines*, Acta Polytechnica Scandinavica, Electrical Engineering Series, no. 110, Espoo 2002, Finish Academies of Technology.

[7] A.A. Abdel-Razek, J.L. Coulomb, M. Feliachi, J.C. Sabonadière, "The calculation of electromagnetic torque in saturated electric machines with combined numerical and analytical solution of the field equations," *IEEE Transactions on Magnetics*, vol. 17, no. 6, pp. 3250-3252, November 1981.

[8] H. W. Derbas, J. M. Williams, A. Koenig, and S. D. Pekarek, "A comparison of nodal- and mesh-based magnetic equivalent circuit models," IEEE Transactions on Energy Conversion, vol. 24, no. 2, pp. 388–396, June 2009.